Calibration of partial safety factors for the structural design of glass

Massimo Badalassi [1], Luigi Biolzi [2], Gianni Royer-Carfagni [3], Walter Salvatore [4]

[1] Dept. of Civil and Industrial Engineering, University of Pisa, Italy (m.badalassi@dic.unipi.it)
[2] Dept. of Architecture, Built Environment and Costructions, Technical School of Milan, Italy (biolzi@stru.polimi.it)
[3] Dept. of Industrial Engineering, University of Parma, Italy (gianni.royer@unipr.it)
[4] Dept. of Civil and Industrial Engineering, University of Pisa, Italy (walter@dic.unipi.it)

Abstract
The semi-probabilistic method necessitates proper partial safety-factors for material strength, calibrated in order to achieve the desired probability of collapse in the construction life-time. Starting from a statistical distribution of glass strength *à là* Weibull, obtained in a previous extensive experimental campaign, here the calibration has been conducted using a full probabilistic method of level III in paradigmatic examples, accounting for wind, snow and personnel (anthropic) actions. Results are in agreement with empirical estimates based upon experience and practice.

1 Introduction

Glass is being more and more used with structural purpose as beams, plates or shells, to form columns, fins, walls, frames, façades, roofs [Beatini & Royer-Carfagni., 2011]. Glass structures are rather expensive and potentially dangerous because of the intrinsic brittleness of the material, but quite surprisingly the current design practice still relies upon rules of thumb or personal experience, sometimes not corroborated by definite structural calculations. This is why there is an increasing effort, both at the national and international level, to define consistent structural codes especially conceived of for the design of glass, according to the same basic concepts and safety requirements used for other construction materials (concrete, steel, timber) when used in structural applications.
However, the design of structures made of glass presents specific peculiarities with respect to others of traditional building materials [Biolzi *et al.*, 2010]. Glass is the brittle material *par* excellence. This renders its use in structural applications quite problematic because even a whatsoever small accident may produce catastrophic collapse. In fact, whereas steel or concrete structures possess sufficient structural ductility to accommodate unusual loading and/or distortions, glass breaks whenever the local value of the stress overcomes the limit value of strength in a whatever small portion [Royer Carfagni & Silvestri, 2009]. Failure of glass is in fact associated with the progression of one dominant defect (micro-crack), i.e., the one which undergoes the most severe combination of stress with respect to its intrinsic size (crack width and stress intensity factor) [Shand, 1961; Wiederhorn & Bolz, 1970; Wan *et al.*, 1961]. This is why the weakest-link model of failure, usually interpreted at the macroscopic level by a Weibull statistical distribution of material strengths, is usually considered the one which best adapts to this case [Evans, 1978; Batdorf & Heinisch,1978; Chao & Shetty, 1990].
In view of the use of the semi-probabilistic, or level I, method for structural calculations [CEN-TC250, 2005a], it is of crucial importance the proper definition of partial safety-factors for material strength, to be calibrated in order to achieve the desired probability of collapse in the construction life-time [Madsen *et al.*, 1985]. However, due to the aforementioned peculiarities of glass, it would be meaningless to use for such coefficients the numbers traditionally employed for other building materials. Also the use of methods of level II should be questioned, because the values of the safety margin (β index) depends upon the specific probability distributions that are used to interpret the effects of actions and the material resistance and, to our

knowledge, no specific treatment does exist for what the Weibull distribution, specifically, is concerned [Madsen *et al.*, 1985].

We are not aware of any existing standard for glass that considers a probabilistic base for the calibration of partial safety factors. Very recently, for example, the CEN/TC129/WG8 has proposed the new prEN16612[1] "Glass in building – Determination of the load resistance of glass panes by calculation and testing" [CEN-TC129-WG8, 2013], which claims to follow the basics of design established in EN 1990 [CEN-TC250a, 2005]. However, the partial safety factors for material strength proposed in prEN16612 are not justified by a probabilistic calculation, or are not reported because left to the national annexes. There are several national standards especially conceived of for specific applications of glass panes (mainly façades), but the design strengths and safety factors therein proposed, as well as the calculation methods [Galuppi & Royer-Carfagni, 2013], are mainly based upon practical experience and rules of thumb.

It may be worth mentioning that, at the European level, the Regulation n. 305/2011[2], repealing Council Directive 89/106/ECC, has laid down harmonized conditions for the marketing of construction products, defining the basic requirements for construction works through seven categories. All structural elements must comply with Basic Requirement n. 1 (BR1) – Mechanical resistance and stability – which is achieved through the Eurocode documents. On the other hand, existing standards for glass, including prEN16612, have been traditionally considered within the framework of Basic Requirement n. 4 (BR4) – Safety and accessibility in use - which is outside BR1. This may be (partially) justifiable if the glass work is an infill panel whose failure does not implies disproportionate risks to the cause that produced it, but the modern daring use of structural glass does not allow neglecting the mechanical resistance and stability requirements. As a matter of fact, our personal opinion is that limited attention has been paid to mechanical strength in existing regulations: this is why, in some cases, reference to documents referring to BR4 to calculate beams, roofs, balustrades and challenging structural façades may be misleading and inconsistent [Galuppi & Royer-Carfagni, 2013].

Most recently, the works for a new Eurocode on structural glass aiming at achieving BR1 have just started, but a few years from now will be necessary to have at least a preliminary draft. An attempt to fill, at least partially, the aforementioned gap has been made in Italy through the National Research Council document CNR-DT210 "Instructions for the design, construction and control of buildings with structural glass elements" [CNR, 2013]. At the time of the present writing this document has not yet been approved, but its major novelty will be the introduction of the probabilistic approach to structural safety.

The aim of this article is to present a proper calibration of material partial safety factors for the structural design of glass with the probabilistic method. This will be achieved by using full probabilistic methods of level III [Madsen *et al.*, 1985] in paradigmatic examples, accounting for wind, snow and personnel (anthropic) actions. Various aspects will necessitate of particular consideration, among which the statistical description of glass strength through a micromechanically motivated model based upon fracture mechanics. Other aspects of particular importance are the effects of edge finishing (seamed, polished or clean cut) and of surface treatments (enameling, serigraphy, coating). A peculiar phenomenon in glass is that the application of long term loading may produce its rupture at stress levels far below the static strength under short-duration actions. For such a phenomenon, usually referred to as static fatigue, reference must be made to a model of the static propagation of an equivalent dominant crack, which evolves in time according to a power-law dependence of the crack tip velocity upon the stress intensity factor. All the issues must be considered here from a statistical point of view. Glass strength will be interpreted through a probabilistic distribution *à là* Weibull, obtained from a previous, extended, experimental campaign [Dall'Igna *et al.*, 2010]. The results of this study furnish the basis for the design of glass structures according to the general concepts established in

---

[1] This draft standard was initially called prEN13474 [CEN-TC129-WG8, 2012], but although being under inquiry for more than ten years, it was never approved. Because of this, the proponents were forced to withdraw it. After having changed its name to prEN 16612, the procedure of public inquiry has started again.

[2] Approved by the European Parliament on March 9th 2011.

EN 1990 [CEN-TC250a, 2005], and will be probably adopted in the forthcoming Italian recommendations CNR-DT210 [CNR, 2013].

The plan of the article is as follows. In section 2, general concepts for the probabilistic approach to the mechanical resistance and stability are briefly recalled, emphasizing the peculiarities of glass. The probabilistic model of glass resistance that has been used in the calculations is described in detail in Section 3. The procedure for the calibration of the safety factor for annealed glass structures is described in Section 4 and achieved in Section 5, through consideration with a full probabilistic approach of level III of paradigmatic case studies. Of course, the work is far from being exhaustive. Other important problems, such as the statistical characterization of the strength of pre-stressed glass (heat and/or chemically tempered), or the influence of edge finishing on glass strength, could not be dealt with now because the experimental data are still missing. The open issues, with concluding remarks, are summarized in the final Section.

2 Probabilistic approach to stability of glass structures

In any kind of structural work, a certain level of stability and safety against failure is required. Such a level is assessed on a statistical basis, defining the probability of collapse that is reputed acceptable as a function of the *consequences* of the collapse itself and the nominal lifetime of the construction. Such an approach is well codified in European regulation EN 1990 [CEN-TC250, 2005a], but it must be detailed and extended to the specific case of cases of glass structures, which present noteworthy peculiarities.

2.1 Classes of consequences and probability of failure for glass structural elements

EN 1990 defines three classes, referred to as CC1 and CC2 and CC3, according to the potential consequences of the failure of a structure in economic, social, and environmental terms, including loss of human life. Each class is moreover associated with different categories of constructions based on their importance: for example, CC1 refers to agricultural buildings, CC2 to residential and office buildings, CC3 to grandstands and open buildings. Such classification, however, considers the structure in its entirety, i.e., collapse implies loss of the entire construction.

Indeed, because of their cost, elements made of glass are widely used in valuable public buildings. On the other hand, glass structures often represent localized parts of the construction (façades, beams, parapets, staircases, etc.): their failure can certainly have very serious consequences, though hardly ever accompanied by collapse of the entire buildings. Their classification ought therefore to be based on the severity of the potential consequences due to *localized* failure of the element in question, without having necessarily to extend the higher consequence classes to all the glass elements making up the construction. If this was not done, the class CC1 could never be used because glass is hardly employed in agricultural buildings; but upgrading all the glass elements to the higher classes would be uneconomical because there might be elements (the paradigm is window panels), whose collapse has a low risk of loss of human life and modest or negligible economic, social and environmental consequences.

A possible classification, which will be probably adopted by a recent proposal of Italian recommendations [CNR, 2013], is represented in Table 1, which for the sake of completeness includes, apart from the three classes set forth in EN 1990, also a class CC0, which refers to all *strictly non-structural* elements. The assessment of the safety level for class CC0, which includes for example standard window glass panes, can be obtained on the basis of specific tests and practical design rules, but does not require in general a specific structural design likewise the elements that fall within the higher classes.

Table 1. Proposal of classes of consequences for glass elements, according to their specific importance.

Class	*Definition*
CC0	Specifically non-structural elements. Following failure, negligible economic, social and environmental consequences and practically null risk of loss of human life.
CC1	Following failure, low risk of loss of human life and modest or negligible economic, social and environmental consequences. Glass structural elements whose failure involves scarce consequences fall into to this category.
CC2	Following failure, moderate risk of loss of human life, considerable economic, social and environmental consequences. Glass structural elements whose failure involves medium-level consequences belong to this category.
CC3	High risk of loss of human life, serious economic, social and environmental consequences: for instance, the structures of public buildings, stages and covered galleries, where the consequences of failure can be catastrophic (concert halls, crowded commercial centers, etc.). Glass structural elements whose failure involves high-level consequences fall into this category.

Glass structural elements can thus be divided into the following classes:
Class zero: elements with no structural function, with a consequence class CC0;
First class: elements with a consequence class CC1;
Second class: elements with a consequence class CC2;
Third class: elements with a consequence class CC3.

Each class of structural element can generally be assigned a decreasing probability of collapse, from the zero class to the third class, as they correspond to ever-more significant and serious consequences as a function of the design lifetime of the structure in question. The design or nominal lifetime of a structure or a structural element in general refers to the period during which the structure is assumed suitable for use, with programmed maintenance, but without the need for substantial repair operations. The reference values of design lifetime for various types of civil constructions are recorded in EN 1990 [CEN-TC250, 2005a]. Once the construction's category, and hence the design lifetime is established, it is possible to assign for each class of glass structural element the corresponding probabilities of collapse, which for the first, second and third classes are assumed to be equal to those indicated in the EN 1990 standard as recorded in Table 2.

Table 2. Probability of collapse as a function of the different structural element classes.

Class	*Probability of collapse*
Zero	probability of collapse to be evaluated in consideration of costs of maintenance and repair
First	$\mathbf{4.83 \cdot x\ 10^{-4}}$ over 50 years; $\mathbf{1.335 \cdot 10^{-5}}$ in 1 year.
Second	$\mathbf{7.235 \cdot x\ 10^{-5}}$ over 50 years; $\mathbf{1.301 \cdot 10^{-6}}$ in 1 year.
Third	$\mathbf{8.54 \cdot x\ 10^{-6}}$ over 50 years; $\mathbf{9.960 \cdot 10^{-8}}$ in 1 year

As regards class zero, the probability of collapse should be decided on the basis of experience, also bearing in mind the costs associated with possible replacement. In any case, its evaluation should not be within the tasks of a structural code.

2.2 Probabilistic procedures for safety level evaluation

In the semi-probabilistic method, classified as the level I method in EN 1990, partial amplifying factors for the actions and partial reduction factors for the strengths are used, so to conduct the safety check through direct comparison of weighted stresses and strength values. For its proper use, partial safety factors must be calibrated in such a way that in a probabilistic view of safety, such comparison is indicative of the performance levels required for the construction in terms of the probability of failure.

As a rule, the numerical values for the partial safety factors for the material checks can be determined in one of the two following ways: performing calibration *a*) based on experience and building traditions or *b*) based on statistical evaluation of the experimental data and field observations. Here we will use approach *b*), implemented within a micromechanically-motivated model for evaluation of structural reliability. The partial safety factors of the different materials will be calibrated in such a way that the reliability levels of representative structures are as near as possible to the preset reliability index.

In the calibration procedure, full probabilistic methods, also called Level III methods, will be employed. These should be preferred to methods of level II [Madsen *et al.*, 1985], because the latters have been proved to provide reliable results for most structural applications with traditional materials (steel, concrete, wood), but their use for the specific case of glass is yet to be verified. In fact, glass is certainly an innovative material from the structural standpoint, also considering that the random variable associated to its resistance value follows a Weibull probability density law for which level II methods, calibrated *via* the statistical distributions of traditional materials, appear less reliable. Level III methods are instead more thorough, in that they involve direct evaluation of the probability of failure based upon the statistical distributions of the actions acting on the construction and the strengths of the materials.

In the probabilistic approach, the measure of reliability must be equated with the probability of survival $P_s = (1 - P_f)$, where P_f *is* the probability of failure for the considered collapse mode or limit state, calculated for a certain reference period. If the calculated probability of failure is greater than a preset objective value, then the structure is to be considered unsafe.

Denoting by S a synthetic symbol indicating the domain of the acting forces, let $f_S(s)$ indicate the probability distribution law of the values $s \in$ S. By analogy, denoting with R the resistance domain, let $f_R(r)$ be the probability distribution law for $r \in$ R. The *performance function* G(R,S) identifies the safe zone of the plane (R, S) as G > 0, and the zone corresponding to failure as G < 0. The probability of failure P_f can therefore be determined, based on the probability distribution laws of $r \in$ R and $s \in$ S, as the probability of occurrence of the condition G(R,S) ≤ 0, or in summary form, $P_f = P$[G(R,S) ≤ 0]. For cases in which R and S are independent variables, G(R,S) = R – S and we obtain the expression

$$P_f = P\left[R - S \leq 0\right] = \int_{-\infty}^{+\infty} \int_{-\infty}^{s \geq r} f_R(r) \cdot f_S(s) dr\, ds \,. \tag{2.1}$$

Moreover, in cases for which the resistance and force domains are made to coincide (through the use of a proper structural model), then $r \equiv s \equiv x$, $x \in$ X, and we obtain

$$P_f = P\left[R - S \leq 0\right] = \int_{-\infty}^{+\infty} F_R(x) f_S(x) dx \;\;, \tag{2.2}$$

where $F_R(x)$ represents the cumulative distribution function of the resistances.

3 Probabilistic model of glass resistance

Since the mechanical resistance of glass depends essentially on the presence of superficial cracks of random size and orientation, experimental data generally turn out to be broadly dispersed and require a statistical basis for interpretation, which must rely upon a micromechanically-motivated model.

3.1 Subcritical crack growth

Fracture of quasi-brittle materials is in general caused by propagation of one dominant crack in mode I, being negligible the contributions in mode II and mode III [Brückner-Foit et al., 1996]. In general, the most critical cracks are the superficial ones that, at least as a first order approximation, can be assumed to be semielliptical. In this case, the stress intensity factor is given by

$$K_I = \sigma_\perp Y \sqrt{\pi c} \,, \tag{3.1}$$

where $\sigma_\perp$ is the (macroscopic) component of stress normal to the crack plane, c is the length of the open axis of the ellipses and Y is a coefficient that takes into account the geometry of the crack (for a semicircular crack this is of the order of 0.71 [Murakami, 1987]). The material strength in general depends upon the size of the largest (or critical) defect in a specimen, so that the study can be restricted to a dominant crack. If f_c represents the measured (macroscopic) stress at collapse, acting at right angle to the dominant crack, the corresponding critical size of the crack is given by

$$c_c = \left(\frac{K_{IC}}{Y f_c \sqrt{\pi}} \right)^2 , \tag{3.2}$$

where K_{IC} is the critical stress intensity factor that depends upon the material fracture toughness and, for ordinary float glass is of the order of K_{IC} = 0.75 MPa $m^{1/2}$ [Wiederhorn, 1969].

A noteworthy peculiarity of glass is that in general cracks progress in time when their size is well below the critical limit c_c. Such a phenomenon of *subcritical crack growth* is usually referred to a *static fatigue*. It is reasonable to assume that the speed of subcritical crack growth is a function of the stress intensity factor. For the case of brittle materials and for glass in particular, similarly to cyclic fatigue [Choi et. al., 2006] it is customary to assume a power law of the type [Evans, 1972]

$$\frac{dc}{dt} = v_0 \left(\frac{K_I}{K_{IC}} \right)^n = v_0 \left(\frac{\sigma_\perp Y \sqrt{\pi c}}{K_{IC}} \right)^n , \tag{3.3}$$

where v_0 and n are material parameters that depend upon the type of glass. For soda-lime glass v_0 varies between 30 μm/s in dry air to 0.02 mm/s in water, but in general it is assumed v_0 = 0.0025 mm/s in any condition [Porter & Houlsby, 1999]. The exponent n varies between 12 and 16 according to the hygrometric conditions, but it can be prudentially assumed n = 16 for 100% humidity (for borosilicate glass $n = 27 \div 40$).

Of course, expression (3.3) is representative of an intermediate asymptotic phase when $K_0 < K_I < K_{IC}$, i.e., K_I is higher than an activation threshold K_0 but lower than the critical value K_{IC}, in proximity of which the power law (3.3) is not experimentally verified. However, one can remain on the side of safeness by neglecting the threshold K_0; moreover, the critical stage of crack growth ($K_I \cong K_{IC}$) is so rapid that (3.3) is usually assumed to be valid also when $0 < K_I \le K_{IC}$. In a load history defined by $\sigma_\perp = \sigma(t)$, integration between the origin $t = 0$, when the crack length c is initially c_i, and the collapse stage $t = t_f$, when $c = c_c$, gives

$$\int_{c_i}^{c_c} c^{-\frac{n}{2}} dc = \int_0^{t_f} v_0 \left(\frac{\sigma\ t\ Y\sqrt{\pi}}{K_{IC}} \right)^n dt \,. \tag{3.4}$$

In general, experiments are controlled at constant stress rate, i.e., $\sigma(t) = \dot{\sigma}\, t$ with $\dot{\sigma}$ constant. Therefore, denoting with f_{test} the tensile strength measured at the end of the stress-driven test, integrating (3.4) and recalling (3.2) for $f_c = f_{test}$ and that $f_{test} = \dot{\sigma}\, t_f$, being t_f the time of rupture, one finds

$$c_i = \left[\frac{n-2}{2} \frac{v_0}{n+1} \left(\frac{Y\sqrt{\pi}}{K_{IC}} \right)^n \frac{f_{test}{}^{n+1}}{\dot{\sigma}} + \left(\frac{Y f_{test} \sqrt{\pi}}{K_{IC}} \right)^{n-2} \right]^{\frac{2}{2-n}} \cong \left[\frac{n-2}{2} \frac{v_0}{n+1} \left(\frac{Y\sqrt{\pi}}{K_{IC}} \right)^n \frac{f_{test}{}^{n+1}}{\dot{\sigma}} \right]^{\frac{2}{2-n}} . \tag{3.5}$$

The approximation is due to the fact that, as it can be verified a posteriori, the second term inside the square parentheses is negligible with respect to the first one. The quantity c_i results to be an intrinsic material parameter, representative of the defects that are initially present in the material. Such a parameter characterizes, through the proposed modeling, not only the macroscopic strength of the material, but also its attitude to the phenomenon of static fatigue.

Observe in passing that the most noteworthy consequence of (3.5) is that, since c_i is a material parameter, the quantity $(f_{test})^{n+1}/\dot{\sigma}$ is a constant. We will thus make the position

$$R = \frac{(f_{test})^{n+1}}{\dot{\sigma}}, \tag{3.6}$$

where the constant R may be measured from experimental tests, and for soda-lime float glass may be assumed of the order of 7.2 10^{22} $\mathrm{MPa}^n\cdot\mathrm{s}$.

Such finding furnishes the rescaling for the measured strength [Collini & Royer-Carfagni, 2013] when tests are performed at different load speed, and it is the basis for the experimental assessment of the coefficient n, as also indicated in ASTM C1368 [ASTM, 2001].

3.2 Load duration and the coefficient k_{mod}

In the design practice, actions are usually schematized by loads assumed to remain constant for a characteristic time, representative of their cumulative effect during the construction life-time. Referring to (3.1), let us suppose that the load produces a (macroscopic) stress $\sigma_\perp = \sigma_L$ at the crack tip in the characteristic interval $0 \le t \le t_L$, being $t = t_L$ the time when failure occurs. Denoting with c_{cL} the corresponding critical crack size, then (3.4) becomes

$$\int_{c_i}^{c_{cL}} c^{-\frac{n}{2}} dc = \int_0^{t_L} v_0 \left(\frac{Y \sigma_L \sqrt{\pi}}{K_{IC}} \right)^n dt\,. \tag{3.7}$$

After integration, one obtains

$$\sigma_L{}^n t_L = \frac{\dfrac{2}{n-2} c_i^{\frac{2-n}{2}} \left[1 - \left(\dfrac{c_i}{c_{cL}} \right)^{\frac{n-2}{2}} \right]}{v_0 \left(\dfrac{Y\sqrt{\pi}}{K_{IC}} \right)^n}\,. \tag{3.8}$$

Since in general $c_i << c_{cL}$, as it can be verified *a-posteriori*, recalling expression (3.5) and the definition of R from (3.6), this expression can be simplified in the form

$$\sigma_L{}^n t_L \cong \frac{\dfrac{2}{n-2} c_i^{\frac{2-n}{2}}}{v_0 \left(\dfrac{Y\sqrt{\pi}}{K_{IC}} \right)^n} = \frac{1}{n+1} R\,. \tag{3.9}$$

From this, one can predict the stress σ_L that produces failure in a time t_L, i.e.,

$$\sigma_L = \left(\frac{1}{n+1}\right)^{1/n} \left(\frac{t_L}{R}\right)^{-1/n} .$$

(3.10)

A practical way to consider this effect is through the coefficient k_{mod}, defined as

$$k_{\text{mod}} = \frac{\sigma_L}{f_{ref}} = \frac{1}{f_{ref}} \left(\frac{1}{n+1}\right)^{1/n} \left(\frac{t_L}{R}\right)^{-1/n}$$

(3.11)

where f_{ref} is a reference value for the failure stress. In general, it is customary to assume f_{ref} to be the bending strength in a text at constant load rate $\dot{\sigma}_{ref}$ = 2MPa/s. Since clearly $(f_{ref})^{n+1} = R\,\dot{\sigma}_{ref}$, one obtains

$$k_{\text{mod}} = \left(\frac{1}{n+1}\right)^{1/n} R^{\frac{1}{n(n+1)}} \; \dot{\sigma}_{ref}^{\frac{-1}{n+1}} \; t_L^{-\frac{1}{n}} .$$

(3.12)

For float glass, setting n = 16 and assuming characteristic values for the other parameters, one obtains $k_{mod} = 0.9759\,(t_L)^{-1/16}$, where t_L is measured in seconds. Observe that this expression is slightly different from that recorded in prEN 16612 [CEN-TC129-WG8, 2013], but since no motivation, either experimental or theoretical, is contained in that document, in the following (3.12) will be used. In any case, the difference with the expression of prEN 16612 is minimal.

3.3 Statistical distribution of defects

A two-dimensional solid (i.e., a plate), with mid-surface area A, can be considered to be composed of a large number of elements of area dA, each of which is characterized by internal defects of a certain size. Failure due to the application of external loads occurs when any element of area dA fails (*weakest link model*). The probability of an element's failure is therefore linked to the probability that that particular element be called upon to contain a critical defect.

By likening any defects existing on the surface to cracks orthogonal to the surface itself, it is convenient to define the tensile resistance of the glass, not in reference to the stress intensity factor, but rather to the mean stress calculable in a hypothetical defect-free element. It can thus be assumed that the fracture propagates when the stress component normal to the plane of the crack exceeds the critical value σ_{Ic}, which represents the mean maximum uniaxial stress in an element with one crack along the direction orthogonal to the stress axis (mode I) in the absence of static fatigue. In general, the size, density and orientation of cracks on the surface of a solid can be evaluated through probabilistic laws. According to the Weibull formulation, the mean number of cracks in a unit area with mechanical resistance below σ_{Ic} can be expressed in the form [Evans, 1978; Batdorf & Heinisch,1978; Chao & Shetty, 1990]

$$N(\sigma_{Ic}) = \left(\frac{\sigma_{Ic}}{\eta_0}\right)^m$$

(3.13)

where the parameters here generically indicated with m (modulus) and η_0 (reference resistance) depend on the material fracture toughness and the statistical distribution of the surface crack size. An high value of m indicates a low dispersion of the mechanical resistances, corresponding to a homogeneous defectiveness of the test specimen. For $m \to \infty$ the range of resistance has amplitude tending to 0, and all the elements have the same mechanical resistance.

3.4 Statistical effects of stress state and specimen size

Since the leading crack opening mechanism is in mode I, the probability that the maximum principal tensile stress is at right angle to the crack plane is higher when the state of stress is biaxial rather than uniaxial [Brückner-Foit et al., 1996]. Moreover, the probability of failure also depends upon the size of the area that is subjected to tensile stress. Assuming that defects are uniformly distributed (all directions have the same probability of having a dominant crack), for a plane state of stress the failure probability can be assumed of the form [Munz & Fett, 1999]

$$P = 1 - \exp\left\{ -\int_A \left[\frac{1}{\pi} \int_0^{\pi} \left(\frac{\sigma_\perp}{\eta_0} \right)^m d\varphi \right] dA \right\},$$

(3.14)

where $\sigma_\perp$ is the normal component of stress in the direction of the angle φ and η_0 is a parameter having the dimension of a stress times unit-area to the $1/m$ exponent (MPa $mm^{2/m}$). If σ_1 and σ_2 ($\sigma_1 > \sigma_2 > 0$) are the principal component of stress, making the direction $\varphi = 0$ coincide with the principal direction σ_1 of maximal tensile stress, introducing the variable $r = \sigma_2/\sigma_1$, expression (3.14) can be written in the form

$$P = 1 - \exp\left\{ -\int_A \left[\left(\frac{\sigma_1}{\eta_0} \right)^m \frac{1}{\pi} \int_0^{\pi} \left(\cos^2\varphi + r\,\sin^2\varphi \right)^m d\varphi \right] dA \right\}.$$

(3.15)

Following [Munz & Fett, 1999], one can thus introduce the correction factor for the state of stress

$$C = \left[\frac{2}{\pi} \int_0^{\pi/2} \left(\cos^2\varphi + r \cdot \sin^2\varphi \right)^m d\varphi \right]^{1/m}$$

(3.16)

and write (3.14) in the form

$$P = 1 - \exp\left[-\int_A \left(\frac{C\sigma_1}{\eta_0} \right)^m dA \right] = 1 - \exp\left[-kA \left(\frac{\sigma_{max}}{\eta_0} \right)^m \right],$$

(3.17)

where σ_{max} denotes the maximum tensile stress in the loaded area, and we have introduced the quantity kA ($k < 1$), referred to as the *effective area*, defined as

$$kA = \left(\sigma_{max} \right)^{-m} \int_A \left(C\sigma_1 \right)^m dA.$$

(3.18)

In order to compare different-in-type experimental results, it is necessary to assume a reference test condition and to rescale all the data with respect to this. It is customary to assume, as the reference, the distribution corresponding to an ideal *equi-biaxial* stress distribution acting on the *unitary* area UA =1 m^2, i.e., $\sigma_1 = \sigma_2 = \sigma_{eqb,UA}$. For this case, C =1 in (3.18), so that (3.17) reads

$$P_{eqbiax} = 1 - \exp\left[-UA \left(\frac{\sigma_{eqb,UA}}{\eta_0} \right)^m \right].$$

(3.19)

The probability of failure at the stress σ_{max} given by (3.17) is equal to the probability of failure at the stress

$\sigma_{eqb,UA}$ given by (3.19) provided that

$$UA\ \sigma_{eqb,UA}{}^{m} = kA\ \sigma_{\max}{}^{m} \quad \Rightarrow \quad \sigma_{eqb,UA} = \sigma_{\max}\left(\frac{kA}{UA}\right)^{1/m}$$

(3.20)

In words, this expression allows to rescale the experimental values to a common denominator represented by the ideal equi-biaxial stress distribution on the unitary area.

3.5 Statistical effect of load duration

Because of the phenomenon of static fatigue referred to in Section 3.2, it can be of interest to define the re-scaling of the probability distribution (3.19) when, instead of rupture under standard test conditions, one is interest in rupture occurring in the characteristic time t_L, defined by the duration of the applied action. Let m_L and η_{0L} represent the Weibull parameters associated with the corresponding probabilistic distribution, which will have an expression identical to (3.19) with the substitution $m \rightarrow m_L$ and $\eta_0 \rightarrow \eta_{0L}$, i.e.,

$$P_{eqbiax,L} = 1 - \exp\left[-UA\left(\frac{\sigma_{eqb,UA,L}}{\eta_{0L}}\right)^{m_L}\right].$$

(3.21)

The failure probability is the same provided that

$$\left(\frac{\sigma_{eq,UA}}{\eta_0}\right)^{m} = \left(\frac{\sigma_{eq,UA,L}}{\eta_{0L}}\right)^{m_L}.$$

(3.22)

But with a procedure analogous to that leading to (3.9) one has

$$(\sigma_{eq,UA,L})^{n} t_L = \frac{1}{n+1}\frac{(\sigma_{eq,UA})^{n+1}}{\dot{\sigma}}.$$

(3.23)

Inserting (3.23) into (3.22), we obtain

$$\left(\frac{\sigma_{eq,UA}}{\eta_0}\right)^{m} = (\sigma_{eq,UA})^{\frac{m_L(n+1)}{n}}\frac{1}{(\eta_{0L})^{m_L}}\left(\frac{1}{(n+1)\dot{\sigma}\,t_L}\right)^{\frac{m_L}{n}},$$

(3.24)

whence it follows that

$$\frac{m_L(n+1)}{n} = m, \quad \frac{1}{\eta_0^{m}} = \frac{1}{\eta_{0L}^{m_L}}\left(\frac{1}{(n+1)\dot{\sigma}\,t_L}\right)^{\frac{m_L}{n}}.$$

(3.25)

From this we can derive

$$m_L = \frac{n}{n+1}m, \quad \eta_{0L} = \eta_0^{\frac{m}{m_L}}\left(\frac{1}{(n+1)\dot{\sigma}\,t_L}\right)^{\frac{1}{n}} = \eta_0^{\frac{n+1}{n}}\left(\frac{1}{(n+1)\dot{\sigma}\,t_L}\right)^{\frac{1}{n}},$$

(3.26)

which enables converting the statistical distributions of the standard test and to the statistical distributions for a constant load applied for a fixed time period.

3.6 Experimental data

In general, the strength of glass is defined based on standardized tests according to EN 1288, under precise temperature and humidity conditions [T = 23°C, RH = 55%] and at constant loading rate [$\dot{\sigma} = 2\text{MPa}/s$]. An extensive experimental campaign aiming at the statistical characterization of glass strength was conducted in Italy by Stazione Sperimentale del Vetro, and the relevant results recorded in [Dall'Igna *et al.*, 2010]. The tests were Four-Point-Bending (FPB) tests according EN 1288-3 [CEN-TC129, 2000b] and Concentring Double Ring (CDR) tests according to EN 1288-2 [CEN-TC129, 2000a].

The second type of tests [CEN-TC129, 2000a] aims at achieving a perfectly equibiaxial state of stress in the specimen also by imposing an overpressure on the specimen surface. Since this technique is quite complicated, the procedure was slightly modified, avoiding to add the overpressure but exactly calculating the state of stress (close to, but not exactly, equibiaxial) and correcting the experimental data by calculating the correction factor C from (3.16) and the effective area kA from (3.18). For this procedure, more details can be found in [Dall'Igna *et al.*, 2010]. Results from all the experiments were then rescaled according to (3.20) in order to refer them all to the ideal case of the equibiaxial state of stress acting on the unit area.

A noteworthy result of the aforementioned experimental campaign was that there is a substantial difference between the results that are obtained when it is the tin-side, or the air-side, surface that undergoes the tensile stress. Recall that during the float production process, one side of the floating glass paste is directly in contact with the molten tin bath (tin surface), whereas the other surface is directly exposed to air (air surface). This is why, in general, it is necessary to consider two distinct strength distributions for the two sides. This distinction influences the calibration of the partial safety factors, as it will be shown later on.

In conclusion, the probabilistic model that will be used to interpret the strength of float glass is interpreted by a distribution à là Weibull as per (3.19), where the parameters m and η_0, from the experimental results of [Dall'Igna *et al.*, 2010], are summarized in Table 3.

Table 3. Weibull parameters of the surface of annealed float glass plates of thickness 6 mm obtained from experimental tests. Mechanical resistance data referred to a unit surface (UA = 1 m^2).

stressed surface	m	η_0 [MPa mm$^{2/m}$]
Tin	7.3	406
Air	5.4	1096

Recall that such Weibull parameters are associated with the ideal reference test for a unitary area subjected to an equibiaxial state of stress and correspond to the case of tests at standard load rate $\dot{\sigma} = 2\,\text{MPa/s}$. However, in general the design actions (permanent loads, wind, snow) act for the whole structure lifetime, so that phenomena of static fatigue become of importance. We may assume that the characteristic load of each action is constantly applied for an effective *characteristic* time, which represents the time in which the action under consideration produces in the structure lifetime the same damage if it was constant and equal to its characteristic value. In other words, the characteristic time t is equivalent to the integral of the spectrum. Reference value for the effective characteristic load duration are given in table Table 4.

Table 4. Nominal values of the effective characteristic load duration.

Action	Load spectrum		Time *t* equivalent to the integral of the spectrum
	Characteristic reference value	Nature	
Wind	mean over 3 sec.	Maximum peak pressure	3/5 sec
	Mean over 10 min.	Peaks of pressure repeated	15 min.
Snow	Yearly maximum		3 months
Live service load (maintenance)	Brief	Single peak of load	30s
Crowd loading	Brief	Single peak of load	30 s
Crowd loading	Yearly maximum	Repeated loads	12 hours
Daily temperature variation	Maximum daily difference	Duration of the maximum peak	11 hours
Permanent actions			
Self-weight and other dead loads	Permanent	Load Invariable over time	Nominal lifetime

Therefore, while evaluating the probability of collapse, one should consider the Weibull distribution of glass strength corresponding to load constantly acting for their characteristic duration. The corresponding Weibull parameters m_L and η_{0L}, can be found from the characteristic load duration of Table 3 according to the rescaling indicated by (3.26). On the other hand, the characteristic value of glass strength needs to be modified with the coefficient k_{mod} that, as illustrated in Section 3.2, can be evaluated through the expression (3.12).

4 Safety factors for annealed glass structures

The procedures are now described for the calibration of partial safety factors, by comparing the results obtainable with methods of Level III and Level I in some paradigmatic cases. A new method is also proposed for the specialization of the verifications to elements belonging to the various classes of consequences described in Section 2.1, which is based on the introduction of the coefficient R_M. Since in general the greatest majority by far of glass elements can be thought of falling in either first or second class, the calibration will be detailed for these categories only. In general, glass elements falling in third class represent such exceptional structures that, for their design, specific considerations and full probabilistic approaches are envisaged. This is why the calibration will not be detailed for structures falling in the third class, although the result could be directly achieved using the same procedure hereafter detailed.

4.1 Calibration procedure for safety factors

With reference to a glass plate under a certain combination of actions, described by probabilistic models, once the statistical distribution of the actions is given, it is possible to calculate the cumulative probability of the maximum stress in the element. To this end, let us indicate by $F_{\sigma.pr,t}(x)$ the probability that the maximum stress in the plate due to that action, with characteristic duration t, be below the value x in the reference time, assumed here to be one year.
The probability density function of the stresses $f_{\sigma,pr,t}$ can clearly be obtained by derivation with respect to x, that is

$$f_{\sigma.pr,t}(x) = \frac{d}{dx} F_{\sigma.pr,t}(x) \ .$$

(4.1)

With regard to the resistance of the glass, in order to determine parameter k in (3.18) that calibrates the area A of the plate under study in order to define the effective area $A_{eff} = k\ A$, the representative domain of the glass surface subjected to tensions is divided into N idealized elementary areas. Then the mean value of the principal stress components $\sigma_{1,i}$ and $\sigma_{2,i}$ and the ratio $r_i = \sigma_{1,i}/\ \sigma_{2,i}$ is considered for the i-th element, $i = 1,..,N$, and coefficient $C = C_i$ thus calculated *via* (3.16) by placing $r = r_i$. Denoting ΔA_i as the area of the i-th element of the division, from (3.17) the probability of the plate's failing under the given load condition can be approximated *via* the expression

$$P = 1 - \exp\left[-\sum_{i=1}^{N} \left(\frac{C_i\ \sigma_{1,i}}{\eta_0} \right)^m \Delta A_i \right] ,$$

(4.2)

and consequently the counterpart of (3.18) reads

$$k = \frac{\sum_{i=1}^{N} \left(C_i\ \sigma_{1,i} \right)^m \Delta A_i}{A \left(\sigma_{\max} \right)^m} .$$

(4.3)

The value of k therefore depends in general on the coefficient m, but not on η_0.

It has been outlined in Section 3.6 that the "tin" and "air" faces of glass plies are characterized by different in type Weibull distributions. Since the glass arrangement is wholly random, the tin side and the air side have the same probability of being the face subjected to the maximum tensile stresses. To account for the equal probability of these two *incompatible* events, the probability function to be considered is the arithmetic average of the probability functions (4.2) calculated for the two surfaces. These probability functions have the parameter values σ_{0L} and m that, beginning with the data in Table 3, are calculated from (3.26) by setting $t_L = t$, i.e., the characteristic duration time of action as indicated in Table 4. We therefore indicate with

$$F_{\sigma.A.t}^{(air+tin)/2}\ (x) = 1 - \frac{1}{2} \left\{ \exp\left[-k_{air}\ A \left(\frac{x}{\eta_{0L.air}} \right)^{m_{L.air}} \right] + \exp\left[-k_{tin} A \left(\frac{x}{\eta_{0L.tin}} \right)^{m_{L.tin}} \right] \right\} \ ,$$

(4.4)

the cumulative probability of the plate undergoing failure due to maximum stresses below the value x in the reference time period, assumed here to be one year.

The probability of plate collapse in one year of life is obtained through the convolution integral

$$P_{f,1y} = \int_{-\infty}^{+\infty} F_{\sigma.A.t}^{(air+tin)/2}(x) \cdot f_{\sigma,pr,t}(x) \cdot dx$$

(4.5)

To obtain in this expression the rated value defined in Section 2.1 for elements in the various classes of consequences, the characteristic parameters that define the action (e.g. the characteristic wind pressure value) are made to vary until the desired value is obtained.

At this point, we move on to plate design via level I methods. The characteristic values of the actions that produce the target probability of collapse, multiplied by suitable partial coefficients γQ, are used as deterministic values for calculating the maximum stress $\sigma_{\max,d,t}$ in the glass.

Regarding the characteristic resistance of the glass, the value for comparison is represented by $f_{g,k}$, referring to the characteristic resistance obtained in equi-biaxial double ring tests with overpressure according to EN

1288-2 [CEN-TC129, 2000a], whence A_{test} = 0.24 m^2, k_{test} = 1 and $A_{eff.\text{test}}$ = 0.24m^2. It is unanimously assumed $f_{g,k}$ = 45 MPa for float glass.

4.2 The R_M coefficient

In general, partial safety factors are calibrated to achieve a probability of failure associated to the second class, as recalled in Table 2. In order to pass from verifications from the second class to the first class, the EN 1990 [CEN TC250, 2005a] prescribes to reduce the partial coefficient of the actions through the coefficient $K_{FI} < 1$. For statistical distributions of the Gaussian type, Section B3.3 of EN 1990 suggests to use $K_{FI} = 0.9$. However, the case of glass is substantially different[3] because the probabilistic distribution of strength is of the Weibull type. Moreover, since glass is extremely brittle and its distribution of strength quite dispersed, even a small increase of the accepted probability of collapse would provide a substantial decrease of the coefficient K_{FI}. The forthcoming calculations indicate that this should be of the order of $K_{FI} = 0.7$.

From a physical point of view, to define new probability scenarios, it is not the action that should be rescaled, by the resistance of the material. In fact, the action is what it is, whereas the probability of collapse is strongly influenced by the material strength. If the structural response was linear, there is no difference in rescaling the action of the resistance. But since glass structures are quite slender, second order effects are very important: if the action was reduced through a coefficient K_{FI} of the order of 0.7, second order effect might be underestimated.

This is why, the proposed method for verifications in various classes is based upon a rescaling of the material strength, according to an expression of the type

$$S\left(\gamma_Q Q\right) \leq \frac{f_g}{R_M \gamma_M}\,, \tag{4.6}$$

where $S(\gamma_Q Q)$ indicates the stress induced by the action Q multiplied by the corresponding partial safety factor γ_Q, while f_g is generically representative of glass strength and γ_M is the material partial safety factor. The quantity R_M is a new parameter that takes into account, from a probabilistic point of view, the passage from verifications in various classes. Since in general the factors γ_M will be calibrated for verifications in second class, we will set $R_M = 1$ for this case, whereas $R_M < 1$ characterizes verifications in first class.

4.3 Relationship with methods of level I

More specifically, the verification according to methods of level I will be performed according to an expression of the type

$$\sigma_{\max,d,t} \leq \frac{k_{\text{mod},t}\ \lambda_{gA\text{test}\rightarrow kA}^{(air+tin)/2}\, f_{g,k}}{R_M \gamma_M}\,, \tag{4.7}$$

where $\sigma_{\text{max,d},t}$ represents the stress associated with the considered action of characteristic duration time t, while $k_{\text{mod},t}$ suitably reduces the tensile resistance of the glass to account for the phenomenon of static fatigue as defined in (3.12). For what the characteristic resistance value of the glass $f_{g,k}$ is concerned, as already

[3] Although the method proposed by Section B3.3 of EN 1990 cannot be transferred to glass for the reasons here discussed, it should be mentioned that, quite surprisingly, the project standard PrEN 16612 [CEN-TC129-WG8, 2013], ex PrEN13474 [CEN-TC129-WG8, 2012], prescribes the use of this method. The proposed procedure, however, is not corroborated by any scientific reference.

mentioned this is conventionally assumed equal to the nominal value 45 MPa, corresponding to the value of the standardized tests producing an equi-biaxial state of stress on an area A_{test} = 0.24 m^2 with speed $\dot{\sigma} = 2$ MPa/s, according to EN 1288-2 [CEN TC129, 2000a].

The coefficient $\lambda_{g\text{Atest}\to kA}^{(air+tin)/2}$ enables calibrating the characteristic resistance value, obtained according to EN 1288-2 tests on an area A_{test} = 0.24 m^2, in comparison to the effective area $A_{eff} = kA$ of the case study according to (3.20). Assuming (4.4), i.e., the equal probability of fracture on different stress fields acting on the area of the case study and on the test area (under equi-biaxial conditions), the rescaling (3.26) to account for the characteristic duration of applied load furnishes expressions in (4.4) of the form

$$\begin{aligned}&\exp\left[-k_{air}\,A\left(\frac{\sigma_{\max.A}}{\sigma_{0L.air}}\right)^{m_{L.air}}\right]+\exp\left[-k_{tin}A\left(\frac{\sigma_{\max.A}}{\sigma_{0L.tin}}\right)^{m_{L.tin}}\right]\\&=\exp\left[-1\,A_{test}\left(\frac{\sigma_{\max.test}}{\sigma_{0L.air}}\right)^{m_{L.air}}\right]+\exp\left[-1A_{test}\left(\frac{\sigma_{\max.test}}{\sigma_{0L.tin}}\right)^{m_{L.stagno}}\right].\end{aligned} \tag{4.8}$$

We obtain the corresponding coefficients from the preceding expression by setting $\sigma_{\text{max.test}} = f_{g,k}$ and $\lambda_{g\text{Atest}\to kA}^{(air+tin)/2} f_{g,k} = \sigma_{\max.A}$. However, as the arguments of the exponential are small, if we go on to develop $e^x = 1 + x + o(x)$ in series, and neglect the terms of order beyond the first, such expression reduces to

$$k_{air}\,A\left(\frac{\lambda_{gAtest\to kA}^{(air+tin)/2} f_{g,k}}{\sigma_{0L.aria}}\right)^{m_{L.air}}+k_{tin}A\left(\frac{\lambda_{gAtest\to kA}^{(air+tin)/2} f_{g,k}}{\sigma_{0L.tin}}\right)^{m_{L.tin}}=A_{test}\left[\left(\frac{f_{g,k}}{\sigma_{0L.air}}\right)^{m_{L.air}}+\left(\frac{f_{g,k}}{\sigma_{0L.tin}}\right)^{m_{L.tin}}\right]. \tag{4.9}$$

This expression, although rigorous, is however of little use in that it is not reversible analytically.

An approximate expression can be obtained by separately evaluating the fracture probabilities of the air and tin surfaces, and evaluating the scale effects on the arithmetic average of the rescaled stresses. If the exposed surface subjected to tension was the air side or the tin side, from (3.20) we would respectively get

$$\begin{aligned}\sigma_{\max.A}^{air}&=\sigma_{\max,Atest}\left(\frac{A_{test}}{k_{air}\;A}\right)^{1/m_{L.air}}\;\Rightarrow\;\lambda_{gAtest\to kA}^{air}\;f_{g,k}=f_{g,k}\left(\frac{A_{test}}{k_{air}\;A}\right)^{1/m_{L.air}},\\\sigma_{\max.A}^{tin}&=\sigma_{\max,Atest}\left(\frac{A_{test}}{k_{tin}\;A}\right)^{1/m_{L.tin}}\;\Rightarrow\;\lambda_{gAtest\to kA}^{tin}\;f_{g,k}=f_{g,k}\left(\frac{A_{test}}{k_{tin}\;A}\right)^{1/m_{L.tin}}.\end{aligned} \tag{4.10}$$

Then by setting

$$\sigma_{\max.A}^{(air+tin)/2}:=\frac{1}{2}(\sigma_{\max.A}^{air}+\sigma_{\max.A}^{tin})=:\lambda_{gAtest\to kA}^{(air+tin)/2} f_{g,k}\,, \tag{4.11}$$

we obtain in conclusion

$$\lambda_{g\text{Atest}\to kA}^{(air+tin)/2}=\frac{1}{2}\left[\left(\frac{A_{\text{test}}}{k_{air}A}\right)^{1/m_{L\,air}}+\left(\frac{A_{\text{test}}}{k_{tin}A}\right)^{1/m_{L\,tin}}\right] \tag{4.12}$$

In general, given the high value of the exponents $m_{L.air}$ and $m_{L.tin}$, (4.9) and (4.12) lead to expressions only slightly different from each other. Indeed, as can be verified directly in the case studies presented in the

following, the difference between the values obtained through the two expressions is less than one percentage point.
It lastly should be noted that, in strictly probabilistic terms, the value of $f_{g,k}$ to consider in (4.7) for calibrating the partial safety factor γ_M should be the characteristic value of the in resistances associated to distribution (4.4). In design, the nominal value $f_{g,k}$ = 45 MPa is always used, so that it is preferable to refer to this value in calculations.
Once factor γ_M has been determined for elements in the second class by setting R_M =1, we can thus proceed to analyze the case of first class elements. In this case, coefficient R_M in expression (4.7) is the one that suitably reformulates the resistance values so that they correspond to different collapse probabilities. Using the value of γ_M determined previously, the wanted coefficient R_M is that which furnishes equality in (4.7) corresponding to the target collapse probability for elements in the first class.

5 Case studies

The procedure just described is now applied to some paradigmatic case studies.

5.1 Plate under wind load

Let us consider a monolithic glass plate of dimensions 1000 × 1000 × 6 mm^3, simply supported at the edges, under wind pressure p_w. Two checks are conducted with regard to this action for peak winds acting over different characteristic times of 3 seconds and 10 minutes, because due to the phenomenon of static fatigue, a lower pressure acting for a longer time may be more dangerous than a peak action.
Using a numerical code, the stresses that develop in the plate have been evaluated both in the linear elastic range, and in the geometric non-linear regime, while maintaining constitutive linearity. Figure 1 shows the results obtained in terms of the maximum stresses attained at the center of the plate. It is evident that neglecting the non-linearity can lead to errors in the highest values of p_w. For the case in question, best fitting the results with a second order polynomial reveals that the stress σ_{max} [MPa] can be approximated as a function of p_w [daN/m^2] in the form

$$\sigma_{\max} = -6 \cdot 10^{-5} p_w^2 + 0.0836 p_w =: S(p_w) \,. \tag{5.1}$$

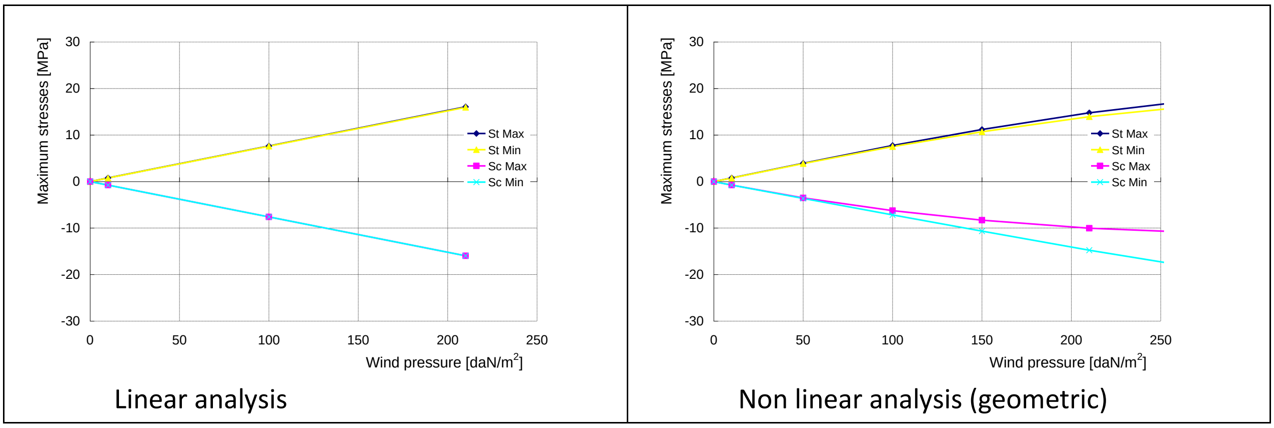


Figure 1. Maximum stress at the plate center as a function of the wind pressure; a) linear elastic analysis; b) elastic analysis with geometric non-linearity.

For meaningful values of p_w (5.1) can easily be inverted to obtain the wind pressure that produces a certain maximum stress, yielding the relation $p_w = S^{-1}(\sigma_{max})$.

The probabilistic model of the wind action to use in the proposed procedure has been obtained by starting with the definitions and rules furnished in the Eurocode 1, EN 1991-1-4 [CEN TC250, 2005b], and also followed by the Italian Circolare Esplicativa alle Norme Tecniche per le Costruzioni [M.I.T., 2009].
In particular, by means of the expression furnished at point C.3.3.2 of [M.I.T., 2009], once the reference velocity $v_{b.50}$ (defined as the characteristic wind velocity value at 10 m above ground on an exposure category II field averaged over 10 minutes) has been determined for a return period of 50 years (assigned by regulations), it is possible to evaluate the reference velocity corresponding to a different return period T_R via an expression of the following type

$$v_b\left(T_R\right)=\alpha_R v_{b.50} \quad , \quad \alpha_R = 0.75\sqrt{1-0.2\cdot\ln\left[-\ln\left(1-\frac{1}{T_R}\right)\right]} \ . \tag{5.2}$$

An analogous expression is set forth in point 4.2(2)P of EN 1991-1-4 [CEN TC250, 2005b], according to which the coefficient c_{prob}, which when multiplied by reference velocity $v_{b.50}$ furnishes the velocity value with an exceedance probability in any 1 year equal to p, can be evaluated through the expression

$$c_{prob}=\left(\frac{1-K\cdot\ln\left(-\ln\left(1-p\right)\right)}{1-K\cdot\ln\left(-\ln\left(0.98\right)\right)}\right)^n \ , \tag{5.3}$$

where K is a shape coefficient that depends on the variation coefficient of the distribution of extreme values. Assigning to K and n the values recommended by the Eurocode, i.e., 0.2 and 0.5 respectively, and substituting probability p with the inverse of the return period value ($p = 1/T_R$), yields the above expression (5.2) for the coefficient α_R.
Since p is the probability of exceeding value v_b in 1 year, the value $(1 - p)$ is the probability that such value not be exceeded in 1 year, which is precisely the sought for cumulative distribution function $F(v_b)$. In conclusion, from (5.2) it is possible to obtain the cumulative distribution function of v_b, the maximum averaged wind velocity over 10 minutes recorded in one year, which is

$$F(v_b)=\exp\left[-\exp\left(\frac{1}{0.2}-\frac{v_b^2}{0.2\cdot 0.75^2\cdot v_{b.50}^2}\right)\right] \ . \tag{5.4}$$

For what the wind pressure is concerned, both EN 1991 [CEN TC250, 2005b] and [M.I.T., 2009] allow to distinguish the peak pressure corresponding to the averaged over time t = 10 min or t = 3 s through expression of the type

$$p_{w,10\min}=\frac{1}{2}\rho\, v_b^2\, c_{e1}(z)\, c_p\, c_d \ , \qquad p_{w,3\mathrm{s}}=\frac{1}{2}\rho\, v_b^2\, c_e(z)\, c_p\, c_d \ . \tag{5.5}$$

In these formulas ρ = 1.25 kg/m^3 is the air density, c_p and c_d are the pressure coefficient and the dynamic factor [M.I.T., 2009], respectively, z is the height above ground, whereas $c_{e1}(z)$ and $c_e(z)$ are the exposure factors. The latters are of the form

$$c_{e1}(z)=\left(\ln\left(\frac{z}{z_0}\right)\right)^2\cdot k_r^{\,2}\cdot c_t^{\,2} \text{ , with } z=z_{\min} \text{ for } z\le z_{\min} \ , \tag{5.6}$$

$$c_e(z) = k_r^2\, c_t(z) \ln\left(\frac{z}{z_0}\right)\left[\ln\left(\frac{z}{z_0}\right)\cdot c_t(z) + 7\right],\ \text{with } z = z_{\min} \text{ for } z \le z_{\min}\ ,$$

(5.7)

where z_0 and $z_{\min}$ are reference heights, k_r is a coefficient that depends on the field exposure category where the construction is located, and $c_t(z)$ is the orographic coefficient. Relation (5.4) is a Gumbel distribution function.

Therefore, in terms of the wind pressure $p_{w,t}$ averaged over time t (t = 3 s or t = 10 min), by substituting for v_b in (5.4), we obtain

$$F\left(p_{w,t}\right) = \exp\left[-\exp\left(\frac{1}{0.2} - \frac{2\,p_{w,t}}{\rho\, c_{e,t}\ c_p\ c_d\ 0.2\cdot 0.75^2\ v_{b.50}^2}\right)\right],$$

(5.8)

being

$$c_{e,t} = \begin{cases} c_e \text{ for } t = 3\text{s}\,, \\ c_{e1} \text{ for } t = 10\,\text{min}\,. \end{cases}$$

(5.9)

Now substituting $p_w = S^{-1}(\sigma_{\max})$ into expression (5.8) yields the cumulative distribution function of the maximum stress in the plate consequent to the maximum annual wind pressure, calculated as the mean over the characteristic time interval t. This turns out to be expressed as

$$F_{\sigma,pr,t}(x) = \exp\left[-\exp\left(\frac{1}{0.2} - \frac{2\,S^{-1}(x)}{\rho\, c_{e,t}\ c_p\ c_d\, 0.2\ 0.75^2\ v_{b.50}^2}\right)\right],$$

(5.10)

where x (in MPa) represents the actual maximum stress value, while $S^{-1}(x)$ is the function introduced above, which furnishes the wind pressure in daN/m^2 that produces the maximum stress x in MPa. The stress probability function, $f_{\sigma pr,t}$ is obviously obtained by deriving (5.10) with respect to x, which yields

$$f_{\sigma,pr,t}(x) = -F_{\sigma,pr,t}(x)\,\exp\left(\frac{1}{0.2} - \frac{2S^{-1}(x)}{\rho\, c_{e,t}\ c_p\ c_d\, 0.2\ 0.75^2\ v_{b.50}^2}\right)\frac{2S^{-1}(x)}{\rho\, c_{e,t}\ c_p\ c_d\, 0.2\ 0.75^2\ v_{b.50}^2}\,\frac{d}{dx}S^{-1}(x)\,.$$

(5.11)

For the Weibull distribution of the resistances, we refer to (3.21), in which expression the Weibull parameters to be considered are the values m_L and η_{0L}that defines the probability of breaking occurring in characteristic time t. Such parameters can be determined from (3.26) as a function of the Weibull parameters m and η_0. Table 5 shows such values in terms of unit area obtained by applying (3.26) to the data of Table 3.

Table 5. Weibull Parameters for the test conditions and breakage in characteristic times t, reported in terms of unit area.

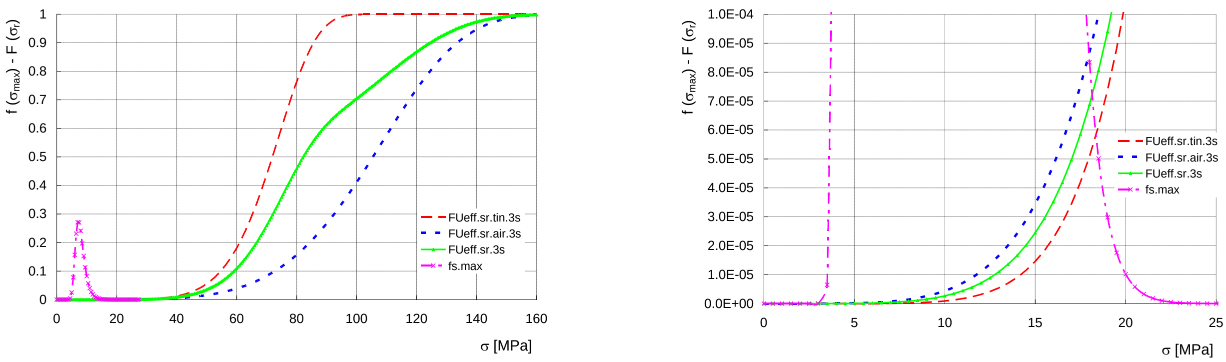
Weibull Parameters		m_L	σ_{0L} [MPa mm$^{2/m_L}$]
Test CDR-UA	Tin	7.3	406
	Air	5.4	1096
t = 3 s CDR-UA	Tin	6.9	425
	Air	5.1	1220

t = 10 min	Tin	6.9	305
CDR-UA	Air	5.1	876

In order to determine parameter k in (4.3), which rescales the plate area A to define the effective area, we hypothetically divide the representative square domain of the glass into N = 400 squares of dimensions 50 × 50 mm, and for the i-th square consider the mean value of the principal stress components $\sigma_{1,i}$ and $\sigma_{2,i}$ and the ratio $r_i = \sigma_{1,i} / \sigma_{2,i}$. We thus calculate coefficient $C = C_i$ defined by integral (3.16), by setting $r = r_i$. With ΔA_i the area of the i-th square of the division, coefficient k is calculated using (4.3).

For the case under examination we obtain k_{air} = 0.1764 and k_{tin} = 0.138. To account for the equal probability that in general either side – air or tin side – be subjected to the greater stress, the probability function considered is the arithmetic average of the probability functions as described by (4.4). The probability of collapse of the plate in one year of life is furnished by (4.5).

It should be noted that, in the case in question, the most meaningful contributions to the convolution integral (4.5) occur in correspondence to the tail of the cumulative distribution function of the resistances, in the interval defined by the probability density function of the actions effects $f_{\sigma,pr,t}$ (Figure 2a). In this portion, magnified in Figure 2b, the cumulative distribution function of the resistances for the air side is greater than the corresponding function for tin side (there is a greater probability of obtaining very small resistance values on the air side than on the tin side). As already anticipated, this confirms that, although the air side is on average more resistant than the tin side, in probabilistic terms, the structural resistance of the latter is actually better than the former.



(a) (b)

Figure 2. a) probability density function of the actions' effects and the cumulative distribution function of the resistances. b) Magnification of the significant portion, with indications of the cumulative resistance distribution function for the air side, the tin side and the average.

Setting in (5.10) and (5.11), without losing generality, $v_{b.50}$ =30 m/s, c_d = 1 and c_p = 1.2, we choose height z of the construction so as to obtain, through the (5.5) and (5.6), the values of coefficients c_{e1} and c_e, for which integral (4.5) furnishes probability $P_{f,1y}$ equal to the objective values established in Table 2, that is $P_{f,1y}$ = $1.335 \cdot 10^{-5}$ for first class elements, and $P_{f,1y}$ = $1{,}301 \cdot 10^{-6}$ for second class elements. Using these coefficient values, the design of the plate in question is optimal.

At this point we proceed to the plate design via level I methods. The *design* wind pressure $p_{w,d,t}$ is obtained from (5.5) by setting $v_b = v_{b.50}$ [CNR, 2008], i.e.,

$$p_{w,10\min}(z) = \frac{1}{2} \cdot \rho \cdot v_{b.50}^2 \cdot c_{e1}(z) \cdot c_p \cdot c_d \ , \tag{5.12}$$

for t = 10 min, and

$$p_{w,3\sec}(z) = \frac{1}{2} \cdot \rho \cdot v_{b.50}{}^2 \cdot c_e(z) \cdot c_p \cdot c_d \ , \tag{5.13}$$

for t = 3 s.

The value of the maximum stress in the plate is obtained by inserting the design wind pressure into (5.1), properly multiplied by the coefficient of the actions γ_Q, which thus yields $\sigma_{\max,d,t} = S(\gamma_Q\, p_{w,d,t})$. Setting $R_M = 1$ for second class checks, the material coefficient γ_M is calculated in such way that yields equality in the following inequality

$$\sigma_{\max,d,t} = S(\gamma_Q \ p_{w,d,t}) \leq \frac{k_{\text{mod},t} \ \lambda_{gAtest\rightarrow A}^{(air+tin)/2} \ f_{g,k}}{R_M \gamma_M} , \tag{5.14}$$

where γ_Q = 1.5, $f_{g,k}$ = 45 MPa, the coefficient $k_{\text{mod},t}$ is calculated from (3.12) as the function of the characteristic time t of load application.. Regarding the coefficient, $\lambda_{g\text{Atest}\rightarrow kA}^{(air+tin)/2}$ using the values in Table 5, from (4.12) we obtain for the case in examination the value

$$\lambda_{g\text{Atest}\rightarrow A}^{(air+tin)/2} = \frac{1}{2}\left[\left(\frac{0.24\,\text{m}^2}{0.176 \cdot 1\,\text{m}^2}\right)^{1/m_{L\,air}} + \left(\frac{0.24\,\text{m}^2}{0.138 \cdot 1\,\text{m}^2}\right)^{1/m_{L\,tin}}\right] = 1.07 \tag{5.15}$$

for both t = 3 sec, and t = 10 min.

For class 1 elements, the same expression is used, but in this case R_M is the coefficient that, bearing in mind the statistical distribution of the actions and resistances, suitably reformulates the resistance values so as to correspond to different collapse probabilities. The value of the coefficient R_M is thus calibrated in such a way as to yield in (5.14) an equality with the same value of γ_M calculated for verifications in second class.

The values obtained for the case at hand are summarized in Table 6.

Table 6. Verifications and partial coefficients for a plate subjected to wind actions.

n°	Class	Design wind	Verification formula	Function of performance probabilistic	$P_{f,1y}$	T_R	R_M	γ_M
1	2	$Q_{w,\max}$ (c_e)	$\sigma_{\max,d,3\sec} \leq \frac{f_{g,k}\lambda_{g\text{Atest}\rightarrow A}^{(air+tin)/2}(k_{\text{mod}}=0.91)}{(R_M=1)\gamma_M}$	$f_{g(CDA;Aeff;3\sec)}^{(air+tin)/2}$	1.30×10^{-6}	50	1	2.56
2	2	$Q_{w,\text{mean}}$ (c_{e1})	$\sigma_{\max,d,10\min} \leq \frac{f_{g,k}\lambda_{gAtest\rightarrow A}^{(air+tin)/2}(k_{\text{mod}}=0.65)}{(R_M=1)\gamma_M}$	$f_{g(CDA;Aeff;10\min)}^{(air+tin)/2}$	1.30×10^{-6}	50	1	2.50

3	1	$Q_{w,\max}$ (c_e)	$\sigma_{\max,d,3\text{sec}} \leq \dfrac{f_{g,k}\lambda_{g\text{Atest}\to A}^{(air+tin)/2}(k_{\text{mod}}=0.90)}{R_M\gamma_M}$	$f_{g(CDA;Aeff;3\text{sec})}^{(air+tin)/2}$	1.33×10^{-5}	50	0.706	2.56
4	1	$Q_{w,\text{mean}}$ (c_{e1})	$\sigma_{\max,d,10\min} \leq \dfrac{f_{g,k}\lambda_{g\text{Atest}\to A}^{(air+tin)/2}(k_{\text{mod}}=0.65)}{R_M\gamma_M}$	$f_{g(CDA;Aeff;10\min)}^{(air+tin)/2}$	1.33×10^{-5}	50	0.683	2.50

In this regard, it should first of all be noted that the resulting value of R_M is of the order of 0.7. We recall that, at point B3.3, EN 1990 prescribes using a reduction coefficient K_{FI} for the actions equal to 0.9. Such a difference in the values is due to the fact that the value suggested by EN 1990 has been calibrated essentially on Gaussian type probabilistic resistance distributions, while the distribution of glass resistances is instead of the Weibull type, with very high data dispersion. Thus, even a small increase in the probability of collapse, may be associated with a considerable reduction in the design actions.

It is important to emphasize that the partial safety factors here found for fist-class structures are not in contrast with those, based mainly on experience and construction traditions, suggested in version 2009 of PrEN 13474. However, PrEn13474 did not account for scale effects, while instead verification (5.14) contains the coefficient $\lambda_{g\text{Atest}\to kA}^{(air+tin)/2}$, which is greater than one for this case.

5.2 Roof under snow load

The procedure is analogous to that illustrated in the foregoing. From the relation (determined via the finite element model) between the uniformly distributed load due to snow and the maximum tensile stress, it is possible to determine the distribution of the maximum stress on a plate under a snow load on the roofs. Then, from the convolution integral between the probability density function of the maximum tensile stress of the plate subjected to roof snow loads and the cumulative distribution function of the glass fracture resistance, it is possible to determine the probability of collapse of the plate. In the present case, an equivalent characteristic duration of the load is assumed to be 1 month.

Let us consider a uniform plate, 1000 × 1000 × 6 mm^3, of second class glass, with a corresponding target fail probability equal to 1.301×10-6 in one year.

The probabilistic model of snow actions used in the proposed procedure for calculating the partial safety factors for glass structures has been obtained by starting with the formula (D.1) for adjusting ground snow loads with varying return periods, as reported in Appendix D of EN 1991 1- 3 [CEN TC250, 2003], and valid under the hypothesis that the distribution of the annual maximum snow loads follows the Gumbel probability distribution function. Such expression is presented in the form

$$q_{sn} = q_{sk}\left\{\frac{1 - V\dfrac{\sqrt{6}}{\pi}\left[\ln\left(-\ln\left(1-P_n\right)\right)+0.57722\right]}{\left(1+2.5923V\right)}\right\}, \tag{5.16}$$

where: q_{sk} is the characteristic snow load value on the ground (with a 50 year return period); q_{sn} is the snow load referred to a return period of n years (therefore according to the annual probability of exceedance, P_n); P_n is the annual probability of exceedance (approximately equivalent to $1/n$, where n is the corresponding return interval (in years)); V is the variation coefficient of the series of maximum annual snow loads.

As P_n is the annual probability of exceedance, then $1- P_n$ is the annual probability of non-exceedance and thus the ordinate of the cumulative distribution function for ground snow loads with a reference period of 1 year. From expression (5.16), we obtain $1- P_n = F(q_{sn})$, which yields precisely the aforesaid cumulative function in the form

$$F_{q_{sn}}(x) = \exp\left[-\exp\left[\left[1 - \frac{x}{q_{sk}}(1+2.5923V)\right]\cdot\frac{\pi}{V\cdot\sqrt{6}} - 0.57722\right]\right]. \tag{5.17}$$

Such an expression depends on parameters q_{sk} and V. The value of q_{sk} is furnished by regulations as a function of the climate zone and the altitude above sea level (a.s.l.). The value of the variation coefficient is supposed to be provided by competent national authorities. EN 1991 1-3 provides a graph in which the variation coefficient is assumed to vary in the range of 0.2 to 0.6.

Once ground snow loads have been established, it is possible to determine the snow loads on the roofs q_s via the expression

$$q_s = \mu_i \; q_{sk} \; C_E \; C_t \,, \tag{5.18}$$

where μ_i is the roof shape coefficient; q_{sk} is the characteristic reference value of ground snow loads for a 50-year return period; C_E is the exposure coefficient, which is a function of the specific orographic characteristics of the area where the construction is located; C_t is a thermal coefficient that accounts for the reduction in snow load due heat transfer from construction. The values of these coefficients can be drawn from national technical regulations [CEN-TC250, 2003].

Now, by substituting (5.18) into (5.17), we obtain the following expression for the distribution of snow loads on roofs

$$F_{q_{sn}}(x) = \exp\left[-\exp\left[\left[\left[1 - \frac{x}{q_{sk}\mu_1\cdot C_E\cdot C_t}(1+2.5923\,V)\right]\cdot\frac{\pi}{V\cdot\sqrt{6}} - 0.57722\right]\right]\right]. \tag{5.19}$$

In a first analysis, we fix the coefficient of variation of snow loads, defined in (5.16) as equal to $V = 0.2$, and then vary the height a.s.l. until the value of the failure probability p_f obtained with the convolution integral is equal to the target value. Maintaining constant height, a deterministic design is then carried out by varying the material partial coefficient until equality is reached in the relationship

$$\sigma_{\max,d,t}\left(\gamma_Q q_{s,d,t}\right) \le \frac{k_{\mathrm{mod},t}\cdot\lambda_{gAtest\to A}^{(air+tin)/2} f_{g,k}}{R_M\gamma_M}\,, \tag{5.20}$$

where $q_{s,d,t}$ is the design snow load, R_M = 1 for second class checks, $k_{\mathrm{mod},t} = 0.388$ relative to a load application duration of t = 1 month. Like for (5.15), once again in this case we obtain a value of coefficient $\lambda_{g\mathrm{Atest}\to kA}^{(air+tin)/2}$ of approximately 1.07.

The procedure has been repeated for first class elements, for which a target probability value of 1.335×10^{-5} was assumed. The height a.s.l. is varied until the obtained value of p_f is equal to the target p_f value. Maintaining constant height, a deterministic design is then carried out by keeping the material partial safety factor constant at the value determined for second class verifications, and then deriving the maximum value R_M admitted by resistance checks (5.20). Analogous calculations have also been repeated for a variation coefficient value of $V = 0.6$. The results obtained are summarized in Table 7.

Table 7. Verifications and partial safety factors for a plate subjected to snow actions. Results for a 1000 × 1000 × 6 mm^3 plate.

$n°$	Class	V	Verification formula	Probabilistic performance function	$P_{f,1y}$	T_R	R_M	γ_M
1	2	0.2	$\sigma_{\max,d,1\mathrm{month}} < \dfrac{f_{g,k} \cdot \lambda_{gAtest\to A}^{(air+tin)/2}(k_{\mathrm{mod}}=0.388)}{(R_M=1)\,\gamma_M}$	$f_{g(CDA;Aeff;1\mathrm{month})}^{(air+tin)/2}$	1.30×10^{-6}	50	1	2.50
2	2	0.6	$\sigma_{\max,d,1\mathrm{month}} < \dfrac{f_{g,k} \cdot \lambda_{gAtest\to A}^{(air+tin)/2}(k_{\mathrm{mod}}=0.388)}{(R_M=1)\,\gamma_M}$	$f_{g(CDA;Aeff;1\mathrm{month})}^{(air+tin)/2}$	1.30×10^{-6}	50	1	2.30
3	1	0.2	$\sigma_{\max,d,1\mathrm{month}} < \dfrac{f_{g,k} \cdot \lambda_{gAtest\to A}^{(air+tin)/2}(k_{\mathrm{mod}}=0.388)}{R_M\gamma_M}$	$f_{g(CDA;Aeff;1\mathrm{month})}^{(air+tin)/2}$	1.33×10^{-5}	50	0.668	2.50
4	1	0.6	$\sigma_{\max,d,1\mathrm{month}} < \dfrac{f_{g,k} \cdot \lambda_{gAtest\to A}^{(air+tin)/2}(k_{\mathrm{mod}}=0.388)}{R_M\gamma_M}$	$f_{g(CDA;Aeff;1\mathrm{month})}^{(air+tin)/2}$	1.33×10^{-5}	50	0.674	2.30

It is noteworthy that that the case of V = 0.2 is in general more conservative than V = 0.6. The resulting partial safety factor values are comparable to those for wind actions, even if slightly less restrictive.

5.3 Floor under live load

Let us now consider the case study of a building floor panel. In evaluating the variable vertical load acting on the floor we refer to the indications furnished for each specific structural category in current national regulations and Eurocode 1 EN 1991 1-1 [CEN-TC250, 2002]. The live loads have to account for uniformly distributed vertical loads, q_k and concentrated vertical loads Q_k. Loads Q_k are particularly important, especially in the case of glass floors, given the brittle nature of the material. For local check calculations, such loads must be viewed as conventional loads. Moreover they cannot be superimposed on the distributed vertical loads, which must instead be used for calculating the global stresses. In the absence of precise indications, the concentrated loads are considered to be applied on an area of 50 × 50 mm.
The characteristic values of the vertical service loads for different buildings categories are those indicated in the relevant current regulations. The actions are generally represented by the characteristic values for a 50-year return period. The live loads present on the floors are caused by the weight of the furniture, equipment, stored objects and people, without including in this type of load the structural and non-structural permanent loads. Differing live loads are anticipated according to the intended use of the building.
Live loads vary randomly in both time and in space. Variations in space are assumed to be homogeneous, while the variations over time are divided into two components: a “permanent” and “discontinuous” one. The first takes into account the furnishings and heavy equipment: small fluctuations in this load are included in the uncertainties. The discontinuous component represents all types of variable loads not covered by the “permanent” component, such as gatherings of people, crowded halls during special events or the piling up of objects during renovations. Both components are modeled as stochastic processes.
The stochastic field representing the load intensity is defined through two independent variables, V and U: the first associated to variations in the mean intensity of the load on the surface, while the second represents

the random spatial distribution of the load on the surface itself.
The "permanent" component is modeled as an uniformly distributed equivalent load, which can be represented via a Poisson process in which the interval between one loading event and the subsequent one is distributed exponentially with an expected value of λ_p. The intensity of the permanent load is assumed to have a Gamma distribution with expected value of μ_p and standard deviation σ_p equal to

$$\sigma_p = \sqrt{\sigma_V{}^2 + \sigma_{U,p}^2 \cdot \kappa \frac{A_0}{A}} \, , \tag{5.21}$$

in which σ_V is the standard deviation of random variable V, while $\sigma_{U,p}$ represents the standard deviation of variable U. Moreover, in this expression, κ is parameter that depends on the influence surface (which for such plates is taken to be equal to 2), A_0 is a reference area that depends on the intended use, while A is the total surface area subjected to the load, with the convention that when $A_0/A > 1$ it is assumed that $A_0/A = 1$. The parameters describing the distribution depend on the use and can be found in [JCSS, 2001].
The "discontinuous" component is also modeled as a Poisson process. The interval between one event and the next is distributed according to an exponential distribution with an expected value of λ_q . The intensity of the "discontinuous" component is assumed to be interpretable via a Gamma distribution with expected value μ_q and standard deviation

$$\sigma_q = \sqrt{\sigma_{U,q}{}^2 \cdot \kappa \frac{A_0}{A}} \, , \tag{5.22}$$

where $\sigma_{U,q}$ is the standard deviation of the stochastic field describing the variability of the distribution of the load on the surface. These parameters, together with the reference interval D_q of the discontinuous load, is furnished in the JCSS Probabilistic Model Code part 2 [JCSS, 2001].
The maximum load is thus obtained for the combined "permanent" and "discontinuous" components, assuming stochastic independence between the two load types. Lastly, the maximum load during a reference period T is obtained by employing the theory of extreme values.
The case study of a glass floor panel finds application in the ceilings of commercial centers and, more generally, buildings open to the public, where people can gather, but which are rarely, if ever, destined to support the "permanent" component of the variable load. Indeed, it is rather unlikely that a glass floor be designed to bear furniture or other furnishings that would defeat the purpose of its transparency, while, on the other hand, it is quite possible that a large number of people gather on the structure. Therefore, regarding the probabilistic model only the "discontinuous" component, regarding loads typical of a commercial centre are considered, assuming a distributed load equal to 5 kN/m^2 for the deterministic design value.
From the relation between the uniformly distributed load and the maximum tensile stress, determined *via* the finite element model, it is possible to find the distribution of the maximum stress in the plate subjected to the variable load. The probability of collapse of the plate subjected to the variable load can be determined from the convolution integral (4.5) between the probability density function of the maximum tensile stress in the plate subjected to the variable load and the cumulative distribution function of the glass fracture resistance.
The distribution function of the glass resistance to fracture is found in a way wholly analogous to the procedure for determining the corresponding function for the plate subjected to wind loads. In the present case, we assume a characteristic load duration, equivalent to the integral of the spectrum, of 12 hours.
In analyzing this case study, as the load is fixed and no parameter is amenable to variations, by which to be able to vary the design load (such as, for example, the height a.s.l. for snow), we vary the geometry, until we obtain a design that leads to a collapse probability value, obtained via the convolution integral, near to target value. From this point, by conducting a deterministic design of the plate itself, it has been possible to vary the material partial safety factor up to the limit value admitted by the resistance check.
Table 8 shows the parameters assumed for the probabilistic load model, in conformity with [JCSS, 2001].

The plate dimensions that yield the optimal design under the design loads are 940 × 940 × 14 mm^3. According to (4.12), such dimensions corresponds to a value of coefficient $\lambda_{gA\text{test}\to kA}^{(air+tin)/2}$ that is still equal to about 1.07.

Table 8. Parameters defining the "discontinuous" component of the variable loads.

Destination of use	A_0 [m^2]	μ_q [kN/m^2]	$\sigma_{U,q}$ [kN/m^2]	λ_q [years]	D_q [days]
Commercial centers and markets susceptible of overcrowding	100	0.4	1.1	1.0	5

The verification is conducted only for second class elements, given that it is inadvisable to classify as first class elements whose collapse can cause people to fall. The values obtained in the case study are shown in Table 9. Note that the order of magnitude of the resulting coefficients γ_M coincide with those for wind actions, shown in Table 6, as well as snow actions, indicated in Table 7.

Table 9. Verifications and partial coefficients for a plate subjected to the action of variable anthropogenic loads. Results for a 940 × 940 × 14 mm^3 plate.

n°	class	Verification formula	Probabilistic performance function	$P_{f,1y}$	T_R	R_M	γ_M
1	2	$\sigma_{\max,d,12h} < \frac{f_{g,k} \cdot \lambda_{gA\text{test}\to A}^{(air+tin)/2}(k_{\text{mod}} = 0.501)}{(R_M = 1)\,\gamma_M}$	$f_{g(CDA;Aeff;12h)}^{(air+tin)/2}$	1.30×10^{-6}	50	1	2.52

6 Concluding remarks and open issues

The calibration of partial safety factors has been conducted in agreement with the basic principles of design established by EN 1990 (Eurocode 0), by performing a full probabilistic analysis with methods of level III in paradigmatic case studies. In particular we have considered monolithic annealed-glass plates under wind, snow and anthropic loads. The main result is that, in order to achieve a probability of collapse compatible with second class elements according to Table 2 (class of consequences CC2 as in Table 1), one should consider a partial coefficient γ_M = 2.5 ÷ 2.55, as results from the less-restrictive values recorded in Table 6, Table 7 and Table 9. It should be remarked that, to this respect, the most severe action is wind load.

In order to pass from verification in second class to first class, i.e., while considering a higher probability of collapse (Table 2), we propose to use a formula of the type (4.6) or (4.7), where the significance of the coefficient R_M has been discussed at length in Section 4.2. Because of its definition, R_M = 1 for verification in second class, whereas one can consider R_M = 0.7 (Table 6, Table 7 and Table 9) for verification in first class. Here we have not provided the values of R_M for verifications in third class essentially for two reasons. First of all, the accepted probability of collapse for third class is so low (see Table 2) that necessitates the extrapolation of the results of strength statistics on the extreme tails of the Weibull distribution, but this passage necessitates of more accurate investigations. In fact, as discussed at length in [Durchholtz *et al.*, 2005], it is in general not straightforward to statistically assess the strength for glass of very poor quality. On the other hand, elements that belongs to the third class are usually associated with very important structural works, which necessitates particular considerations. Albeit tentatively, a possible categorization of structural elements in classes according to their specific use is recorded in Table 10.

Table 10. Classification of structural glass elements according to their specific use.

Type	Class
Vertical elements** continuously restrained at the borders	1/0*
Vertical elements** with point-wise fixing constraints	2/1*
Horizontal roofing **	2
Parapets with fall hazards	2
Fins	2
Floors, bearing beams	2
Pillars, frames (specific studies with level 2 or 3 methods)	3
Notes: (*) Within the same category, the choice of a more or less restrictive class depends on the importance of the work, the risk in case of collapse of the glass, and whether or not immediate safety countermeasures are planned to reduce the consequences of collapse (shoring, shielding, enclosures). (**) An element is considered vertical if its plane forms an angle of less than 15° with the vertical, while any element not satisfying this definition is considered to be horizontal.	

Unfortunately the present study is far from being exhaustive, because apart from the calibration of safety factors for third class structures, there are other issues still open. One of the major unsolved matters is the characterization of the strength of glass borders. In fact, it should be noticed that in all the cases treated in Section 5, the maximal tensile stress are produced approximately at the center of the panel and, in any case, sufficiently far from the borders. There are other typical cases, such as beams or fins, for which the maximum tensile stresses are in the proximity of the edges. Unfortunately, no experimental data are currently available regarding the specific resistance of the edge, nor is the corresponding curve of the distribution of the probabilities of fracture known. Indeed, the mechanical resistance of a plate’s border (neglecting possible chipping produced during handling and installation) depends on the finishing on the edge (cutting, grinding, etc.) and is therefore totally independent of defects on the plate surfaces. In general, mechanical production processes produce homogeneous “damage” on the glass surfaces that tends to lower the average mechanical resistance (low η_0) but reduce its dispersion (high Weibull coefficient m).
Following [Sedlacek et al., 1999], as also described in [Haldimann, 2006], the edge strength depends on the length of the edge itself, rather than on its surface area, in that the critical point is the border between the face and the edge. We may therefore consider the cumulative probability of fracture to be interpretable, analogously to (3.19), *via* a Weibull distribution of the form

$$P_{ed} = 1 - \exp\left[\int_l \left(\frac{\sigma(s)}{\eta_{0.ed}} \right)^{m_{ed}} ds \right], \tag{6.1}$$

where m_{ed} and $\eta_{0.ed}$ represent the Weibull parameters of the distribution, while the integral is assumed to extend the entire length of the tensed edge. Since the stress state on the edge is monoaxial, $\sigma(s)$ in the expression represents the stress at the edge point with coordinate s, whose form is known from the boundary and load conditions. The distribution parameters must be calibrated based on standardized tests, about which, however, there is as yet no unanimous agreement. For example, [Sedlacek et al., 1999] refer to a three-point bending test on beams of length l_{test} = 0.46 m. Thus, assuming the stress along the edge to be linear, (6.1) can be expressed, in a form equivalent to (3.19), as

$$P_{ed} = 1 - \exp\left[l_{eff.test} \left(\frac{\sigma_{\max}}{\eta_{0.ed}} \right)^{m_{ed}} \right] , \quad l_{eff.test} = k_{l.test} l_{test} , \tag{6.2}$$

where l_{test} is the length of the edge for the specimen of reference of the test (l_{test} = 0.46 m), while $l_{eff.test} = k_{l.test}\, l_{test}$ is its effective length. In this expression, $\eta_{0.ed}$ is a parameter with the dimensions of a stress for a length raised to the power of the exponent $1/m_{ed}$.

The calibration of the partial safety factor for calculations in proximity of the edges should then follow the same lines of Section 4, according to the counterpart of expression (4.7) that should read

$$\sigma_{\max,d,t} = S(\gamma_Q\ p_{w,d,t}) \le \frac{k_{\mathrm{mod},t}\ \lambda_{gltest \to l}\ k_{ed}\, f_{g,k}}{R_M \gamma_M} , \tag{6.3}$$

where $k_{mod,t}$ and $f_{g,k}$ are defined as in (4.7). The quantity $\lambda_{gltest \to l}$ is a scaling coefficient that defines the size effect and whose value must be calibrated starting from experimental results.

Unfortunately at the moment there are not enough experimental data for a calibration based upon the statistical distribution of the edge resistance. Other aspects of concern relate to the decrease in resistance due to surface treatments, such as screen printing or etching, for which there are not yet sufficient experimental data. Last but not least, the calibration of partial factors should also include the case of heat or chemically toughened glass, but also for these we are still waiting for sound experimental data that may be processed on a statistical basis.

Acknowledgements

The authors acknowledge partial support of the European Community under grant RFCS-CT-2012-00026, research project “S + G”.

References

ASTM (2001), *ASTM C 1368: Standard test method for determination of slow crack growth parameters of advanced ceramics by constant stress-rate flexural testing at ambient temperature*, American Society Testing of Materials.

Batdorf , S.B. and Heinisch, H. L. (1978), Weakest link theory reformulated for arbitrary fracture criterion, *J. Am. Ceram. Soc.*, **61**, pp. 355-358.

Beatini, V., Royer-Carfagni, G. (2011), Glass as a material, *Materia*, **69**, pp. 36-43.

Biolzi, L., Cattaneo, S., Rosati, G. (2010), Progressive damage and fracture of laminated glass beams, *Construction and Building Materials*, **24**, pp. 577-584.

Brückner-Foit, A., Fett, F., Schirmer, K.S., Munz, D. (1996), Discrimination of multiaxiality criteria using brittle fracture loci, *J. Eur. Ceram. Soc.*, **16**, pp. 1201–1207.

CEN-TC129 (2000a), *EN 1288-2-2000: Glass in building – determination of the bending strength of glass –*

part 2: Coaxial double ring test on flat specimens with large test surface areas, European standard.

CEN-TC129 (2000b), *EN 1288-3-2000: Glass in building – determination of the bending strength of glass – part 3: Test with specimen supported at two points (four point bending)*, European standard.

CEN-TC129-WG8 (2012), *PrEN-13474: Glass in building - Determination of the strength of glass panes by calculation and testing,* Project of European Standard (withdrawn).

CEN-TC129-WG8 (2013), *PrEN-16612: Glass in building- Determination of the load resistance of glass panes by calculation and testing*. Project of European Standard (under inquiry).

CEN-TC250 (2002), *EN 1991-1-1: Eurocode 1 – Actions on structures - Part 1-1: General actions – Densities, self weight, imposed loads on buildings*, European harmonized standard.

CEN-TC250 (2003), *EN 1991-1-3: Eurocode 1 – Actions on structures - Part 1-3: General actions – Snow loads*, European harmonized standard.

CEN-TC250 (2005a), *EN 1990: Eurocode 0 – Basis of structural design*, European harmonized standard.

CEN-TC250 (2005b), *EN 1991-1-4: Eurocode 1 – Actions on structures - Part 1-4: General actions - Wind actions*, European harmonized standard.

Chao, L.Y. and Shetty, D.K. (1990), Equivalence of physically based statistical fracture theories for reliable analysis of ceramics in multi-axial loading, *J. Am. Ceram. Soc.*, **73** (7), pp. 1917-1921.

Choi S.R., Nemeth N.N., Gyekenyesi J.P. (2006), Slow crack growth of brittle materials with exponential crack velocity under cyclic fatigue loading, *Int. J. Fatigue*, **28**, pp. 164–172.

CNR, National Research Council (2013), *CNR-DT210: Istruzioni per la progettazione, l'esecuzione ed il controllo di costruzioni con elementi strutturali di vetro*, Technical document under public inquiry.

CNR, National Research Council (2008), *CNR-DT207: Istruzioni per la valutazione delle azioni e degli effetti del vento sulle costruzioni*, Technical document.

Collini, L., Royer-Carfagni, G. (2013), Flexural strength of glass-ceramic for structural applications, *Eur. J. Cer. Soc.*, submitted.

Dall'Igna, R., D'Este, A., Silvestri, M. (2010), Comments on test methods for determination of structural glass strength, *Proceedings XXV ATIV Conference*, Parma, Italy, pp. 5-13.

Durchholtz, M., Goer, B., Helmich, G. (1995), Method of reproducibility predamaging float glass as a basis to determinate the bending strength, *Glastech. Ber. Glass Sci. Technol.*, **68**, pp. 251-258.

Evans, A.G. (1972), A method for evaluating the time-dependent failure characteristics of brittle materials and its application to polycrystalline alumina, *J. Mat. Science*, **7**, pp. 1137–1146.

Evans, A.G. (1978), A general approach for the statistical analysis of multiaxial fracture, *J. Am. Ceram. Soc.*, **61**, pp. 302-308.

Galuppi, L., Royer-Carfagni, G. (2013), On the inconsistency of a formulation for the effective thickness of laminated glass, recently implemented by standards, *Composites, part B: Engineering*, **50**, pp. 109-118.

Haldimann, M. (2006), F*racture strength of structural glass elements – analytical and numerical modelling, testing and design*, Thesis n. 3671, EPFL Lausanne, CH.

JCSS (2001), *Probabilistic Model Code Part 2: Load Models*, Joint Committee on Structural Safety (JCSS).

Madsen, H.O., Krenk, S., Lind, N.C. (1985), *Methods of structural safety*, Prentice-Hall.

M.I.T., Ministero Infrastrutture e Trasporti (2009), *Nuova Circolare delle norme tecniche per le costruzioni*, Circolare 02/02/2009 n. 617.

Munz, D., Fett, T. (1999), *Ceramics. Mechanical properties, failure behaviour, materials selection*, Springer-Verlag, Heidelberg.

Murakami, Y., (1987), *Stress intensity factors handbook*, Pergamon.

Porter, M.I., Houlsby, G.T. (1999), Development crack size and limit state design methods for edge abraded glass members, *Report N. OUEL 2111/99*, University of Oxford.

Royer-Carfagni, G., Silvestri, M. (2009), Fail-safe point fixing of structural glass. New advances, *Engineering Structures*, **31**, pp. 1661-1676.

Sedlacek, G., Blank, K., Laufs, W., and Güsgen, J. (1999), *Glas im Konstruktiven Ingenieurbau*, Ernst & Sohn, Berlin.

Shand, E.B. (1961), Fracture velocity and fracture energy of glass in the fatigue range, *J. Am. Ceram. Soc.*, **44**, pp. 21-26.

Wan, K.T., Lathabai, S. and Lawn, B.R. (1961), Crack velocity functions and thresholds in brittle solids, *J. Am. Ceram. Soc.*, **44**, pp. 21-26.

Wiederhorn, S.M. (1969), Fracture surface energy of glass, *J. Am. Ceram. Soc.*, **52**, pp. 99–105.

Wiederhorn, S. M., Bolz, L. H. (1970), Stress corrosion and static fatigue of glass, *J. Am. Ceram. Soc.*, **53**, pp. 543-548.