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Effective thickness of laminated glass beams: New expression *via* a variational approach

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ABSTRACT

The performance of laminated glass, which consists of two or more glass plies bonded together by polymeric interlayers, depends upon shear coupling between the plies through the polymer. This is commonly considered by defining the *effective thickness*, i.e., the thickness of a monolithic beam with equivalent bending properties in terms of stress and deflection. General expressions have been proposed on the basis of simplified models by Newmark and Wölfel–Bennison, but they are either difficult to apply or inaccurate. Here, a variational approach to the problem is presented. By choosing appropriate shape functions for the laminated-beam deformation, minimization of the strain energy functional gives new expressions for the effective thickness under any constraint- and load-conditions, embracing the classical formulations as particular cases. Comparisons with numerical experiments confirm the better accuracy of the proposed approach with respect to the previous ones.

1. Introduction

In order to reduce the risk of catastrophic collapse of structures made of glass, the brittle material *par excellence*, an effective technique is to bond two or more glass plies with thermoplastic polymeric interlayers with a treatment in autoclave at high pressure and temperature. This bond is quite strong because it is chemical in type, being due to the union between hydroxyl groups along the polymer and silanol groups on the glass surface. The resulting *laminated glass* is a *safety glass* because, after breakage, the fragments remain attached to the interlayer: risk of injuries is reduced and the element maintains a certain consistency that prevents detachment from fixings. But the interlayer affects also the pre glass-breakage response because it allows the transfer of shear stresses among glass plies, at the price of a relative sliding due to the deformation of the polymer. The assessment of the *degree of connection* offered by the polymer is crucial for the design of glass structures in the serviceability limit state and this is why a great number of studies, including this one, have considered the response of the composite laminated package before first cracking occurs.

Indeed, the polymeric interlayers are too soft to present flexural stiffness *per se*, but they can provide shear stresses that play an important role for the glass-layer interaction [7]. In general, the degree of coupling of two glass layers depends upon the shear stiffness of the polymeric interlayer, as first mentioned by Hooper [3] while studying the bending of simply supported laminated-glass beams. Since then, the problem has been considered by many authors [15], one of the most recent contribution being the careful finite element analysis of [14], which includes an updated list of the most relevant literature.

In the pre glass-breakage modeling no distinction has to be made for what the type of glass is concerned, because all treatments (annealing, heat strengthening, heat or chemical tempering) affect the ultimate strength and the type of rupture (size of resulting shards) but not the elastic moduli (Young's modulus $E \simeq 70$ GPa and Poisson ratio $\nu \simeq 0.2$). If the response of glass is linear elastic up to failure, the response of the polymeric interlayer is highly non-linear, temperature-dependent and viscoelastic. There are three main commercial polymeric films, each one showing peculiar characteristics: Polyvinyl Butyral (PVB), Ethylene Vinyl Acetate (EVA), and Sentry Glass (SG) [16,10]. Pure PVB, a polyvinyl acetate, is stiff and brittle, but addition of softeners imparts plasticity and toughness, though influencing adhesion-strength, elasticity, water-absorbing and dependence on temperature (glass transition temperature T_g of the order of 20–25 °C). Depending on the composition, the properties of EVA, a polyolefine, vary from partial crystalline and thermoplastic to amorphous and rubber-like, but an increased quantity of vinyl acetate improves strength and ultimate elongation, though decreasing melting temperature: when used as interlayers in laminated glass, modified EVAs are employed with mechanical properties similar to PVB. A somehow innovative material is SG, a ionoplast polymer that, when compared with

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